

**STUDENT PROJECT**  
**ON**  
**INDIAN NUMERALS**

**Submitted by**

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# INDIAN NUMERALS

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It is worth beginning this article with the same quote from [Laplace](#) which we give in the article [Overview of Indian mathematics](#). [Laplace](#) wrote:-

*The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. the importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, [Archimedes](#) and [Apollonius](#).*

The purpose of this article is to attempt the difficult task of trying to describe how the Indians developed this ingenious system. We will examine two different aspects of the Indian number systems in this article. First we will examine the way that the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 evolved into the form which we recognise today. Of course it is important to realise that there is still no standard way of writing these numerals. The different fonts on this computer can produce many forms of these numerals which, although recognisable, differ markedly from each other. Many hand-written versions are even hard to recognise.

The second aspect of the Indian number system which we want to investigate here is the place value system which, as [Laplace](#) comments in the quote which we gave at the beginning of this article, seems "so simple that its significance and profound importance is no longer appreciated." We should also note the fact, which is important to both aspects, that the Indian number systems are almost exclusively base 10, as opposed to the Babylonian base 60 systems.

Beginning with the numerals themselves, we certainly know that today's symbols took on forms close to that which they presently have in Europe in the 15<sup>th</sup> century. It was the advent of printing which motivated the standardisation of the symbols. However we must not forget that many countries use symbols today which are quite different from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and unless one learns these symbols they are totally unrecognisable as for example the Greek alphabet is to someone unfamiliar with it.

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| — | = | ≡ |

Brahmi one, two, three

One of the important sources of information which we have about Indian numerals comes from [al-Biruni](#). During the 1020s [al-Biruni](#) made several visits to India. Before he went there [al-Biruni](#) already knew of Indian astronomy and mathematics from Arabic translations of some Sanskrit texts. In India he made a detailed study of Hindu philosophy and he also studied several branches of Indian science and mathematics. [Al-Biruni](#) wrote 27 works on India and on different areas of the Indian sciences. In particular his account of Indian astronomy and mathematics is a valuable contribution to the study of the history of Indian science. Referring to the Indian numerals in a famous book written about 1030 he wrote:-

*Whilst we use letters for calculation according to their numerical value, the Indians do not use letters at all for arithmetic. And just as the shape of the letters that they use for writing is different in different regions of their country, so the numerical symbols vary.*

It is reasonable to ask where the various symbols for numerals which [al-Biruni](#) saw originated. Historians trace them all back to the Brahmi numerals which came into being around the middle of the third century BC. Now these Brahmi numerals were not just symbols for the numbers between 1 and 9. The situation is much more complicated for it was not a place-value system so there were symbols for many more numbers. Also there were no special symbols for 2 and 3, both numbers being constructed from the symbol for

1. Here is the **Brahmi one, two, three.**

|                                         |   |   |   |   |   |   |   |   |
|-----------------------------------------|---|---|---|---|---|---|---|---|
| 1                                       | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| —                                       | = | ≡ | + | h | 4 | 7 | 5 | ? |
| Brahmi numerals around 1st century A.D. |   |   |   |   |   |   |   |   |

There were separate Brahmi symbols for 4, 5, 6, 7, 8, 9 but there were also symbols for 10, 100, 1000, ... as well as 20, 30, 40, ... , 90 and 200, 300, 400, ..., 900.

The Brahmi numerals have been found in inscriptions in caves and on coins in regions near Poona, Bombay, and Uttar Pradesh. Dating these numerals tells us that they were in use over quite a long time span up to the 4<sup>th</sup> century AD. Of course different inscriptions differ somewhat in the style of the symbols.

Here is **one style of the Brahmi numerals.**

We should now look both forward and backward from the appearance of the Brahmi numerals. Moving forward leads to many different forms of numerals but we shall choose to examine only the path which has led to our present day symbols. First, however, we look at a number of different theories concerning the origin of the Brahmi numerals.

There is no problem in understanding the symbols for 1, 2, and 3. However the symbols for 4, ... , 9 appear to us to have no obvious link to the numbers they represent. There have been quite a number of theories put forward by historians over many years as to the origin of these numerals. In [1] Ifrah lists a number of the hypotheses which have been put forward.

1. The Brahmi numerals came from the Indus valley culture of around 2000 BC.
2. The Brahmi numerals came from Aramaean numerals.
3. The Brahmi numerals came from the Karoshthi alphabet.
4. The Brahmi numerals came from the Brahmi alphabet.
5. The Brahmi numerals came from an earlier alphabetic numeral system, possibly due to [Panini](#).
6. The Brahmi numerals came from Egypt.

Basically these hypotheses are of two types. One is that the numerals came from an alphabet in a similar way to the Greek numerals which were the initial letters of the names of the numbers. The second type of hypothesis is that they derive from an earlier number system of the same broad type as Roman numerals. For example the Aramaean numerals of hypothesis 2 are based on I (one) and X (four):

I, II, III, X, IX, IIX, IIIX, XX.

Ifrah examines each of the six hypotheses in turn and rejects them, although one would have to say that in some cases it is more due to lack of positive evidence rather than to negative evidence.

Ifrah proposes a theory of his own in [\[1\]](#), namely that:-

*... the first nine Brahmi numerals constituted the vestiges of an old indigenous numerical notation, where the nine numerals were represented by the corresponding number of vertical lines ... To enable the numerals to be written rapidly, in order to save time, these groups of lines evolved in much the same manner as those of old Egyptian Pharonic numerals. Taking into account the kind of material that was written on in India over the centuries (tree bark or palm leaves) and the limitations of the tools used for writing (calamus or brush), the shape of the numerals became more and more complicated with*

*the numerous ligatures, until the numerals no longer bore any resemblance to the original prototypes.*

|                                        |   |   |   |   |   |   |   |   |
|----------------------------------------|---|---|---|---|---|---|---|---|
| 1                                      | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| —                                      | = | ≡ | ୩ | ୫ | ୬ | ୭ | ୮ | ୯ |
| Gupta numerals around 4th century A.D. |   |   |   |   |   |   |   |   |

|                                      |    |     |      |      |      |      |      |      |
|--------------------------------------|----|-----|------|------|------|------|------|------|
| 1                                    | 2  | 3   | 4    | 5    | 6    | 7    | 8    | 9    |
| I                                    | II | III | IIII | IIII | IIII | IIII | IIII | IIII |
| Ifrah's guess for pre-Brahmi numbers |    |     |      |      |      |      |      |      |

It is a nice theory, and indeed could be true, but there seems to be absolutely no positive evidence in its favour. The idea is that they evolved from:

One might hope for evidence such as discovering numerals somewhere on this evolutionary path. However, it would appear that we will never find convincing proof for the origin of the Brahmi numerals.

If we examine the route which led from the Brahmi numerals to our present symbols (and ignore the many other systems which evolved from the Brahmi numerals) then we next come to the Gupta symbols. The Gupta period is that during which the Gupta dynasty ruled over the Magadha state in northeastern India, and this was from the early 4<sup>th</sup> century AD to the late 6<sup>th</sup> century AD. The Gupta numerals developed from the Brahmi numerals and were spread over large areas by the Gupta empire as they conquered territory.

|                                          |   |   |   |   |   |   |   |   |   |
|------------------------------------------|---|---|---|---|---|---|---|---|---|
| 1                                        | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
| १                                        | २ | ३ | ४ | ५ | ६ | ७ | ८ | ९ | ० |
| Nagari numerals around 11th century A.D. |   |   |   |   |   |   |   |   |   |

The Gupta numerals evolved into the Nagari numerals, sometimes called the Devanagari numerals. This form evolved from the Gupta numerals beginning around the 7<sup>th</sup> century AD and continued to develop from the 11<sup>th</sup> century onward. The name literally means the "writing of the gods" and it was the considered the most beautiful of all the forms which evolved. For example [al-Biruni](#) writes:-

*What we [the Arabs] use for numerals is a selection of the best and most regular figures in India.*

These "most regular figures" which [al-Biruni](#) refers to are the Nagari numerals which had, by his time, been transmitted into the Arab world. The way in which the Indian numerals were spread to the rest of the world between the 7<sup>th</sup> to the 16<sup>th</sup> centuries is examined in detail in [7]. In this paper, however, Gupta claims that Indian numerals had reached Southern Europe by the end of the 5<sup>th</sup> century but his argument is based on the *Geometry* of [Boethius](#) which is now known to be a forgery dating from the first half of the 11<sup>th</sup> century. It would appear extremely unlikely that the Indian numerals reach Europe as early as Gupta suggests.

We now turn to the second aspect of the Indian number system which we want to examine in this article, namely the fact that it was a place-value system with the numerals standing for different values depending on their position relative to the other numerals. Although our place-value system is a direct descendant of the Indian system, we should note straight away that the Indians were not the first to develop such a system. The Babylonians had a place-value system as early as the 19<sup>th</sup> century BC but the Babylonian systems were to base 60. The Indians were the first to develop a base 10 positional system and, considering the date of the Babylonian system, it came very late indeed.

The oldest dated Indian document which contains a number written in the place-value form used today is a legal document dated 346 in the Chhedi calendar which translates to a date in our calendar of 594 AD. This document is a donation charter of Dadda III of Sankheda in the Bharukachcha region. The only problem with it is that some historians claim that the date has been added as a later forgery. Although it was not unusual for such charters to be modified at a later date so that the property to which they referred could be claimed by someone who was not the rightful owner, there seems no conceivable reason to forge the date on this document. Therefore, despite the doubts, we can be fairly sure that this document provides evidence that a place-value system was in use in India by the end of the 6<sup>th</sup> century.

Many other charters have been found which are dated and use of the place-value system for either the date or some other numbers within the text. These include:

1. a donation charter of Dhiniki dated 794 in the Vikrama calendar which translates to a date in our calendar of 737 AD.
2. an inscription of Devendravarman dated 675 in the Shaka calendar which translates to a date in our calendar of 753 AD.
3. a donation charter of Danidurga dated 675 in the Shaka calendar which translates to a date in our calendar of 737 AD.
4. a donation charter of Shankaragana dated 715 in the Shaka calendar which translates to a date in our calendar of 793 AD.
5. a donation charter of Nagbhata dated 872 in the Vikrama calendar which translates to a date in our calendar of 815 AD.
6. an inscription of Bauka dated 894 in the Vikrama calendar which translates to a date in our calendar of 837 AD.

All of these are claimed to be forgeries by some historians but some, or all, may well be genuine.

The first inscription which is dated and is not disputed is the inscription at Gwalior dated 933 in the Vikrama calendar which translates to a date in our calendar of 876 AD. Further details of this inscription is given in the [article on zero](#).

There is indirect evidence that the Indians developed a positional number system as early as the first century AD. The evidence is found from inscriptions which, although not in India, have been found in countries which were assimilating Indian culture. Another source is the [Bakhshali manuscript](#) which contains numbers written in place-value notation. The problem here is the dating of this manuscript, a topic which is examined in detail in our article on the [Bakhshali manuscript](#).

We are left, of course, with asking the question of why the Indians developed such an ingenious number system when the ancient Greeks, for example, did not. A number of theories have been put forward concerning this question. Some historians believe that the Babylonian base 60 place-value system was transmitted to the Indians via the Greeks. We have commented in the [article on zero](#) about Greek astronomers using the Babylonian base 60 place-value system with a symbol o similar to our zero. The theory here is that these ideas were transmitted to the Indians who then combined this with their own base 10 number systems which had existed in India for a very long time.

A second hypothesis is that the idea for place-value in Indian number systems came from the Chinese. In particular the Chinese had pseudo-positional number rods which, it is claimed by some, became the basis of the Indian positional system. This view is put forward by, for example, Lay Yong Lam; see for example [8]. Lam argues that the Chinese system already contained what he calls the:-

*... three essential features of our numeral notation system: (i) nine signs and the concept of zero, (ii) a place value system and (iii) a decimal base.*

A third hypothesis is put forward by Joseph in [2]. His idea is that the place-value in Indian number systems is something which was developed entirely by the Indians. He has an interesting theory as to why the Indians might be pushed into such an idea. The reason, Joseph believes, is due to the Indian fascination with large numbers. [Freudenthal](#) is

another historian of mathematics who supports the theory that the idea came entirely from within India.

To see clearly this early Indian fascination with large numbers, we can take a look at the *Lalitavistara* which is an account of the life of Gautama Buddha. It is hard to date this work since it underwent continuous development over a long period but dating it to around the first or second century AD is reasonable. In *Lalitavistara* Gautama, when he is a young man, is examined on mathematics. He is asked to name all the numerical ranks beyond a koti which is  $10^7$ . He lists the powers of 10 up to  $10^{53}$ . Taking this as a first level he then carries on to a second level and gets eventually to  $10^{421}$ . Gautama's examiner says:-

*You, not I, are the master mathematician.*

It is stories such as this, and many similar ones, which convince Joseph that the fascination of the Indians with large numbers must have driven them to invent a system in which such numbers are easily expressed, namely a place-valued notation. He writes in [2]:-

*The early use of such large numbers eventually led to the adoption of a series of names for successive powers of ten. The importance of these number names cannot be exaggerated. The word-numeral system, later replaced by an alphabetic notation, was the logical outcome of proceeding by multiples of ten. ... The decimal place-value system developed when a decimal scale came to be associated with the value of the places of the numbers arranged left to right or right to left. and this was precisely what happened in India ...*

However, the same story in *Lalitavistara* convinces Kaplan (see [3]) that the Indians' ideas of numbers came from the Greeks, for to him the story is an Indian version of [Archimedes'](#) *Sand-reckoner*. All that we know is that the place-value system of the Indians, however it arose, was transmitted to the Arabs and later into Europe to have, in the words of [Laplace](#), *profound importance* on the development of mathematics.

**References** (11 books/articles)

**Article by:** *J J O'Connor* and *E F Robertson*